Worked Solutions

Edexcel C3 Paper C

1. $\dfrac{1}{x+4} - \dfrac{2(x-1)}{(3x+2)(x+4)}$

$= \dfrac{x+4}{(x+4)(3x+2)} = \dfrac{1}{3x+2}.$ **(4)**

$\dfrac{1}{3x+2} = \dfrac{1}{x-5}$

$x - 5 = 3x + 2 \qquad x = -3\frac{1}{2}.$ **(3)**

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2. (*a*) $4x + 1 = e^2$

$x = \dfrac{1}{4}\left(e^2 - 1\right)$ **(3)**

(*b*) $3e^{2x} - 7e^x + 2 = 0$

$\left(3e^x - 1\right)\left(e^x - 2\right) = 0$

$e^x = \dfrac{1}{3};\ e^x = 2$

$x = -\ln 3,\ x = \ln 2$ **(5)**

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3. (*a*) $f(x) \geq 0,\ g(x) < 8$ **(4)**

(*b*) $(x+4)^2 = 8 - x$

$x^2 + 8x + 16 = 8 - x$

$x^2 + 9x + 8 = 0$

$(x+8)(x+1) = 0$

$x = -8,\ -1,$ not in domain of g. **(4)**

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4. (*a*) $f'(x) = x - \dfrac{4}{x-3}$

(*b*) $f'(x) < 0 \Rightarrow x < \dfrac{4}{x-3}$

$x(x-3) < 4 \quad x > 3$

$x^2 - 3x - 4 < 0$

$(x-4)(x+1) < 0$

$-1 < x < 4$

$\therefore 3 < x < 4$

$x^2 - 3x - 4 > 0$ for $x < 3$

$x < -1$ or $x > 4$

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5. $\tan 75° = \dfrac{\cos(75° - 15°) - \cos(75° + 15°)}{\sin(75° + 15°) - \sin(75° - 15°)}$

$= \dfrac{\cos 60° - \cos 90°}{\sin 90° - \sin 60°}$

$= \dfrac{\frac{1}{2} - 0}{1 - \frac{\sqrt{3}}{2}}$

$= \dfrac{1}{2 - \sqrt{3}} \times \dfrac{2 + \sqrt{3}}{2 + \sqrt{3}}$

$= 2 + \sqrt{3}$

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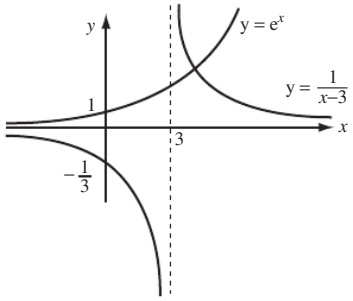
6. (*a*)



(4)

(*b*) curves only cross once. **(2)**

(*c*) $x_1 = 3.04978,\ x_2 = 3.04736,\ x_3 = 3.04748,\ \ldots$

$x = 3.047$ (3 d.p.) **(2)**

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7. (*a*) $x^2 \cdot -3e^{-3x} + e^{-3x} \cdot 2x$

$xe^{-3x}(2 - 3x)$ **(3)**

(*b*) $2\sec x(\sec x \tan x) = 2\sec^2 x \tan x$ **(3)**

(*c*) $\dfrac{5\sin x - 5x\cos x}{\sin^2 x}$ **(3)**

(*d*) $\dfrac{dx}{dy} = -\ln y \cdot \sin y + \cos y \cdot \dfrac{1}{y}$

$\dfrac{dy}{dx} = \dfrac{y}{\cos y - y \ln y \cdot \sin y}$ **(4)**

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8. (*a*) $\dfrac{dy}{dx} = 12\cos 2x - 8\sin 2x$

$\dfrac{d^2y}{dx^2} = -24\sin 2x - 16\cos 2x$

$= -4(6\sin 2x + 4\cos 2x)$

$= -4y.$

(*b*) $R^2 = 6^2 + 4^2$:

$R = \sqrt{52} = 7.211$ (3 d.p.)

$\tan\alpha = \dfrac{2}{3}$

$\alpha = 0.588^c$

(*c*) pt. of inflection

$\Rightarrow \dfrac{d^2y}{dx^2} = 0 \Rightarrow y = 0$

$\Rightarrow \sqrt{52}\sin(2x + 0.588) = 0$

$2x + 0.588 = \pi$

$x = 1.277$(3 d.p.)
